Review Problems for Exam 3

Problem 1. Ramen Republic has a small takeout shop in Simplexville, and is considering expanding its operations. You have been asked to evaluate the current configuration.

The shop has two servers. You have estimated that the time each server spends with a customer is exponentially distributed with a mean of 4 minutes.

You have also estimated that customers arrive outside the shop according to a Poisson process at a rate of 25 per hour. Depending on how many customers are in the shop, a customer may balk and go to Hummus Hipsters next door instead. In particular, if there are i customers in Ramen Republic (including the ones being served), the probability that a customer outside the shop will balk is $\frac{i}{i+1}$ for $i = 0, 1, \dots$. In addition, the shop can hold at most 20 customers, including the ones being served. When the shop is full, any customer arriving outside the shop will go elsewhere.

Model this setting as a birth-death process by specifying the arrival rate and service rate in each state, in terms of the number of customers per hour.

Problem 2. The Simplexville Department of Health and Human Services has recently experienced a significant increase in the number of calls to its hotline. The department has hired you to study how best to staff its call center with its limited resources.

Currently, callers call the hotline at a rate of 6 callers per hour. There are 4 agents working at any given time, and each phone call takes an average of 20 minutes. The phone system does not have a queue, and can only handle 4 callers at a time (4 with an agent) – any phone calls arriving when there are 4 callers in the system are simply turned away with a voice recording.

This setting can be modeled as a birth-death process with the following arrival and service rates:

$$\lambda_i = \begin{cases} 6 & \text{if } i = 0, 1, 2, 3 \\ 0 & \text{if } i = 4, 5, \dots \end{cases} \qquad \mu_i = \begin{cases} 3i & \text{if for } i = 1, 2, 3, 4 \\ 12 & \text{if for } i = 5, 6, \dots \end{cases}$$

a. Over the long run, what is the probability that there are n customers in the system? ($n = 0, 1, 2, \dots$)
b. Over the long run, how often do callers get turned away?
c. Over the long run, what is the expected number of callers in the system?
d. Over the long run, what is the expected time a caller spends in the system?

Problem 3. The Hillier Hotel is evaluating its staffing policies to accommodate a surge in demand for its services. Currently, it has 5 clerks stationed in the lobby. Guests arrive at the lobby at a rate of 90 per hour. The average time for a clerk to serve a patron is 3 minutes.

Model this setting as a M/M/5 queue.

a. What is the probability that there are 10 guests in the lobby?
b. Over the long run, how often is at least 1 clerk busy?
c. Over the long run, what is the expected number of guests waiting in line?
d. Over the long run, what is the expected time a guest waits in line?

Problem 4. You have been hired as a consultant by Four Guys Burgers and Fries to study its operations. Your predecessor left you some notes, stating that the typical configuration of a local franchise has "an M/G/5 queueing system".

a. What distribution do the interarrival times follow in this system?
b. What queueing discipline does this system follow?